Section 3.4

Definition of Concavity: Let f differentiable on an open interval I. The graph of f is **concave upward** on I when f' is increasing on the interval and **concave downward** on I when f' is decreasing on the interval.

Test for Concavity: Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition of Point of Inflection: Let f be a function that is continuous on an open interval, and let c be a point in the interval. If the graph of f has a tangent line at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f when the concavity of f changes from upward to downward (or downward to upward) at the point.

Second Derivative Test: Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, then the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.

1) Determine the open intervals on which the graph of $f(x) = \dfrac{2}{x^2+12}$ is concave upward or downward. Use the table to help you find the intervals.

Interval			
Test Value			
Sign of $f''(x)$			
Conclusion			

2) Determine the open intervals on which the graph of $f(x) = \frac{1}{x^2-1}$ is concave upward or downward. Use the table to help find the intervals.

Interval			
Test Value			
Sign of $f''(x)$			
Conclusion			

3) Determine the points of inflection and discuss the concavity of the graph of
$$f(x) = x^4 + x^3 - 3x^2$$
Use the table to help you.

Interval			
Test Value			
Sign of $f''(x)$			
Conclusion			

4) Find the relative extrema for $f(x) = 3x^5 - 5x^3 + 3$. Use the table to help you.

Point			
Sign of $f''(x)$			
Conclusion			

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz #1, 11, 23, 35, 52